Viscoelastic Damping 101

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As a technical professional, one is often faced with the challenge of reducing unwanted vibration in a structure. The effect of this vibration can range in severity from the actual failure of components due to high-cycle fatigue, to the consumer's perception of poor quality in a product that radiates too much noise or rattles excessively. Whether the failure mode is real or perceived, designers and product development engineers from all industries are working harder than ever to solve these types of noise and vibration issues.

One of the most effective and widely used methods for noise and vibration control is passive damping. Damping is most beneficial when used to reduce the amplitude of dynamic instabilities, or resonances, in a structure.

What is Damping?

Damping is the conversion of mechanical energy of a structure into thermal energy. A structure subject to oscillatory deformation contains a combination of kinetic and potential energy. In the case of real structures, there is also an energy dissipation element per cycle of motion. The amount of energy dissipated is a measure of the structure's damping level.

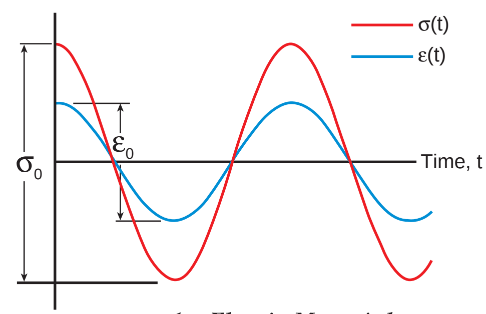
Definition of Viscoelasticity

A viscoelastic material is characterized by possessing both viscous and elastic behavior. What this means exactly is best illustrated in Figure 1, which shows how various types of materials behave in the time domain. For a slab of material with a cross-sectional area, A, and a thickness, T, subject to cyclic loading, F(t), the corresponding response is given by the displacement function, x(t). The cyclic stress on the sample material is found by dividing the input load by the cross-sectional area, and the resulting cyclic strain on the material is found by dividing the displacement by the thickness.

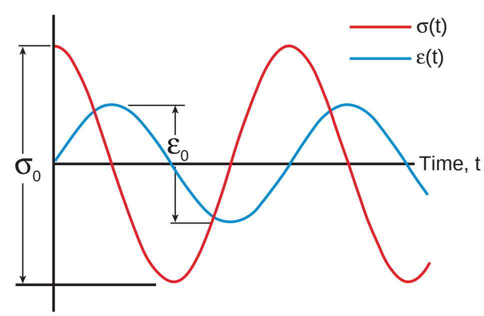
A purely *elastic* material is one in which all the energy stored in the sample during loading is returned when the load is removed. As a result, the stress and strain curves for elastic materials move completely in phase. For elastic materials, Hooke's Law applies, where the stress is proportional to the strain, and the modulus is defined at the ratio of stress to strain.

A complete opposite to an elastic material is a purely *viscous* material, also shown in Figure 1. This type of material does not return any of the energy stored during loading. All the energy is lost as "pure damping" once the load is removed. In this case, the stress is proportional to the rate of the strain, and the ratio of stress to strain rate is known as viscosity, μ. These materials have no stiffness component, only damping.

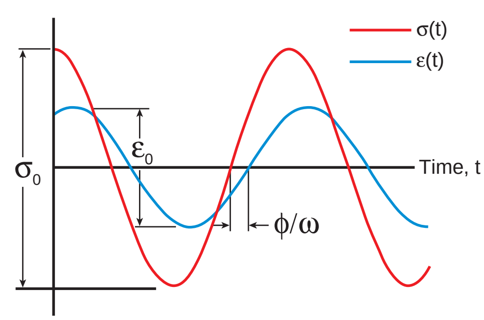
For all others that do not fall into one of the above extreme classifications, we call *viscoelastic* materials. Some of the energy stored in a viscoelastic system is recovered upon removal of the load, and the remainder is dissipated in the form of heat. The cyclic



1a. Elastic Material



1b. Viscous Material



1c. Viscoelastic Material

Figure 1. Cyclic Stress and Strain Curves vs. Time for Various Materials

stress at a loading frequency of ω is out-of-phase with the strain by some angle ϕ, (where $0 < \phi < \pi/2$). The angle ϕ is a measure of the materials damping level; the larger the angle the greater the damping.

For a viscoelastic material, the modulus is represented by a complex quantity. The real part of this complex term (storage modulus, E_1) relates to the elastic behavior of the material, and defines the stiffness. The imaginary component (loss modulus, E_2) relates to the material's viscous behavior, and defines the energy dissipative ability of the material. Using Hooke's Law to define the modulus for complex values, we can define the complex modulus, E* as:

$$E^* = E_1 + E_2 i = \frac{\sigma_0}{\varepsilon_0}\, e^{i\phi}$$

Behavior of Viscoelastic Materials

One of the unique characteristics of viscoelastic materials is that their properties are influenced by many parameters. They can include: frequency, temperature, dynamic strain rate, static pre-load, time effects such as creep and relaxation, aging, and other irreversible effects. In working with this class of materials, we strive to define the materials complex modulus (stiffness and damping properties) as a function of these parameters. Most important of these include temperature and frequency effects. Viscoelastic materials are typically characterized as having the type of behavior as shown in Figure 2.

These materials exist in various unique states or "phases" over the broad temperature and frequency ranges in which they are used. These regions are typically referred to as the Glassy, Transition, Rubbery, and Flow Regions. Viscoelastic materials behave differently based on which region they exist in for a specific application.

In the glassy region the polymer chains are rigidly ordered and crystalline in nature, possessing glass-like behavior. Stiffness, E_1, is at its highest for the material in this region, and damping levels are typically low. The glass transition temperature, T_g, of a material

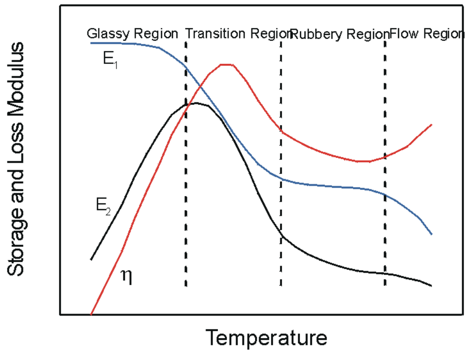
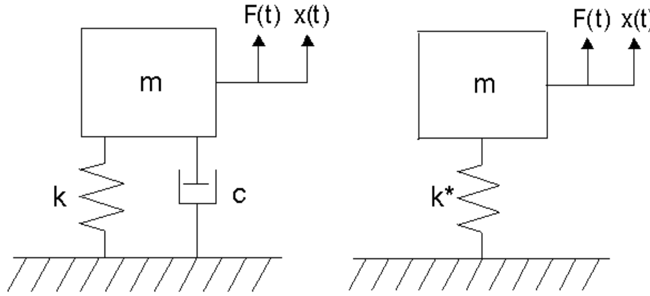


Figure 2. Variation of Complex Modulus with Temperature for a Typical Viscoelastic Material



3a. Viscous Model *3b. Hysteretic Model*

Figure 3. Single-Degree-of-Freedom Damping Models

refers to the elbow of the storage modulus curve at the edge of the glassy region as it enters into the transition region. T_g also defines the peak of the loss modulus, E_Z, curve.

The transition region is so named because the material is transitioning from the glassy to the rubbery region. It is in this area that the viscoelastic material goes through its most rapid rate of change in stiffness and possesses its highest level of damping performance. The reference temperature of a material, T_O, is used to define the peak of the loss factor curve. In this region, the long molecular chains of the polymer are in a semi-rigid and semi-flow state, and are able to rub against adjacent chains. These frictional effects result in the mechanical damping characteristic of viscoelastic materials.

In the rubbery region, the material reaches a lower plateau in stiffness. Damping is at a lower, but reasonable level. A material selected to exist in this region is ideally suited for such devices as isolators or tuned mass dampers because the modulus varies only slightly with changes in temperature and frequency.

Measurement of Damping

The simplified single-degree-of-freedom (SDOF) model shown in Figure 3a can represent many real world structures. In this analytical system, damping is represented using a viscous model in which the damping element (shown as a dashpot) is proportional to velocity, dx/dt. When using a viscous damping model, a direct analytical solution of the equations of motion is available.

The classic complimentary solution to the transient, free vibration response of the system of Figure 3a yields the analytical representation of viscous damping as the damping ratio, ζ, shown in the following equation:

$$\zeta = \frac{c}{c_o} \quad \text{Where: } c_o = 2\sqrt{km} \quad \text{Where: } \%C_r = 100\%\ \zeta$$

The term c_o is the critical viscous damping coefficient. The critical damping for a system is defined as the smallest level of viscous damping in which the mass of Figure 3a will exhibit no oscillation when displaced from equilibrium.

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The damping ratio is often presented as a percentage of the fraction of critical damping or *percent critical damping*, %Cr. A system is classified as *underdamped* if $\zeta < 1$, *critically damped* if $\zeta = 1$, and *overdamped* if $\zeta > 1$. Vibratory motion will only exist for an underdamped system. In all the cases above, the response of a system set into motion will eventually decay to zero with time, except when $\zeta = 0$.

Damping estimates can be made from this transient response. A SDOF underdamped system subjected to an impulsive force at t = 0 will exhibit the displacement response x(t) as shown in Figure 4. A measure of damping can be made from the rate of decay of the response for consecutive cycles of vibration, referred to as the log decrement, δ, and defined in this equation:

$$\delta = \frac{1}{m} \ln \frac{x_{n+m}}{x_n}$$

The selection of a viscous damping model is primarily used for ease of analysis. However, the behavior of a viscoelastic material is better described through the use of the hysteretic model, in which the damping is proportional to strain and is independent of rate. This is achieved by eliminating the viscous

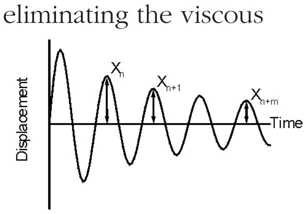

Figure 4. Transient Response of a Classically Underdamped SDOF System

dashpot, and representing the energy dissipation in the system by a complex spring element, k*, resulting in a complex modulus, E*.

The hysteretic model can be used for damping estimates made from the response characteristics of steady-state vibrating systems. The SDOF system of Figure 3b is shown to be subject to a harmonic forcing function, F(t), applied to the mass, m. The equation of motion for this system can be solved to yield the transfer function of the nature shown in Figure 5. This is a graph of compliance for a SDOF system having a single resonance at ω_o. The level of damping can be subjectively determined by noting the sharpness of the peak — the more rounded the shape, the more damping present.

A quantitative measure of damping is achieved by using the *Half-Power Bandwidth method*, shown graphically in Figure 5 and given by the following equation:

$$\eta = \frac{\Delta\omega}{\omega_o} \quad \text{For } n=\sqrt{2} \text{ (Half-Power)}$$

The material damping, η, of the complex spring element can be determined by the ratio of $\Delta\omega$ to ω_o with $\Delta\omega$ determined from the half-power point down from the resonant peak value, Amax. On a decibel scale, this corresponds to –3 dB down from the peak value. For that reason, this damping estimation is also referred to as the *3 dB method*.

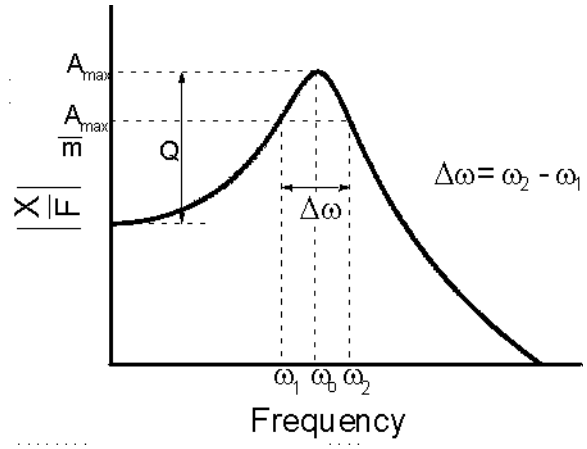

Figure 5. Compliance Transfer Function of a SDOF System

Another representation of damping for this SDOF system is called *Amplification Factor*, Q. As illustrated in Figure 5, Q is the ratio of the response amplitude at resonance, ω_o, to the static response at $\omega = 0$.

Yet another estimate of damping can be achieved by calculating the energy loss per cycle of oscillation due to steady state harmonic loading. For a viscoelastic material subject to the cyclic loading as shown in Figure 1c, the hysteresis of the material can be defined by plotting the input stress σ (t) versus responding strain ε(t) for one cycle of motion. The elliptical shape shown in Figure 6 is defined as the hysteresis loop. The area captured within the hysteresis loop, D, is equal to the dissipated energy per cycle of harmonic motion by the material. For reasonable levels of damping, this relationship between material damping and loop area can be defined by this equation:

$$\eta = \frac{D}{\pi \sigma_o \varepsilon_o}$$

Relationship Between Measures of Damping

For low levels of damping ($\eta < 0.2$), and within the linear region of the viscoelastic material, the different measures of damping discussed above can be equated using the following relationship as shown in Figure 7.

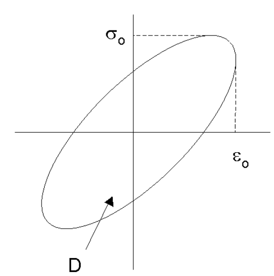

Figure 6. Typical Hysteresis Loop for a Viscoelastic Material

Relationship Between Measures of Damping

$$\eta = \frac{1}{Q} = 2\xi = \frac{\%Cr}{50} = \tan\phi = \frac{\delta}{\pi} = \frac{D}{2\pi U} = \frac{\Delta\omega_{3dB}}{\omega_o}$$

where:

η is loss factor
Q is amplification factor
ξ is damping ratio
%Cr is percent of critical damping
(%Cr = 100% x ξ)
ϕ is the phase angle between cyclic stress and strain
δ is the log decrement of a transient response
D is the energy dissipation per cycle
U is the stored energy during loading

Figure 7. Interrelationship of Damping Measures