Measurement System for Acoustic Absorption Using the Cepstrum Technique

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Abstract
A measurement system is described for acoustic absorption of automotive carpets and materials using the cepstrum technique. The system is based on prior work by Bolton [1] with modern software and hardware implementation. Time-domain averaging, minimum-phase signal generation, late echo removal, and low-frequency data synthesis are implemented using commonly-available commercial hardware and software. The resulting measurement system is capable of making measurements in a normal office environment. Also, the cepstrum technique measurement system can cover a wider frequency spectrum compared to other conventional techniques such as the two-microphone method in an impedance tube or a free field. Results are compared with impedance tube measurements.

1. Introduction
This paper describes a system built to measure the normal-incidence sound absorption of flat material samples using the power cepstrum technique [2]. The system is based on work originally performed by Bolton [1]. The power-cepstrum-based system described here is novel because it was designed to operate in a quiet office environment rather than a hemi-anechoic chamber and because the hardware implementation is inexpensive compared to a dynamic analyzer.

Normal-incidence-sound-absorption measurements are traditionally made using an impedance tube. Several methods are used, but in each case the measured absorption may differ from the true value because the sample is constrained at the tube wall or an air gap exists at the tube wall [3]. The measurement method described here utilizes a large material sample, so there is no edge-constraint effect.

The power-cepstrum measurement system is also capable of making oblique-angle absorption
measurements.

The power-cepstrum method does have limitations. The method is highly susceptible to noise; the method is highly susceptible to errors at low frequency; and the method is most successfully implemented with high-quality audio equipment (loudspeaker, microphone, and microphone amplifier). Steps taken to minimize these issues are described.

2. Power-Cepstrum Theory

Air is assumed to be isotropic and homogeneous. It is further assumed that air is non-dissipative and non-dispersive. The sound pressure at a microphone as shown in Figure 1 can be expressed as:

\[ p(t) = p_1(t) + \frac{r_1}{r_2} h(t) p_1(t - \tau), \]

where \( p_1(t) \) is the direct signal, \( t \) is time, \( r_1 \) and \( r_2 \) are the path lengths as shown in Figure 1, \( \tau \) is the time delay associated with the difference in path lengths, and \( h(t) \) is the impulse response of the reflection process.

In the frequency domain, equation (1) becomes:

\[ P(\omega) = P_1(\omega) + \frac{r_1}{r_2} R(\omega) P_1(\omega) e^{-j\omega \tau}, \]

where \( \omega \) is the frequency in radians per second, \( R(\omega) \) is the reflection coefficient as a function of frequency, and \( j \) is the square root of minus one.

Multiplying by the complex conjugate yields:

\[ |P(\omega)|^2 = |P_1(\omega)|^2 \left[ 1 + \frac{r_1}{r_2} R(\omega) e^{-j\omega \tau} \right] \left[ 1 + \frac{r_1}{r_2} R^*(\omega) e^{j\omega \tau} \right]. \]

Taking the natural logarithm gives:

\[ \ln|P(\omega)|^2 = \ln|P_1(\omega)|^2 + \ln\left[ 1 + \frac{r_1}{r_2} R(\omega) e^{-j\omega \tau} \right] + \ln\left[ 1 + \frac{r_1}{r_2} R^*(\omega) e^{j\omega \tau} \right]. \]

Making use of the series expansion:

\[ \ln(1 + z) = z - \frac{z^2}{2} + \frac{z^3}{3} - \cdots, \]

where \( z \) is any complex variable with magnitude less than one, yields:

\[ \ln|P(\omega)|^2 = \ln|P_1(\omega)|^2 + \frac{r_1}{r_2} R(\omega) e^{-j\omega \tau} + \frac{r_1}{r_2} R^*(\omega) e^{j\omega \tau} - \frac{1}{2} \left( \frac{r_1}{r_2} \right)^2 |R(\omega)|^2 e^{-2j\omega \tau} - \frac{1}{2} \left( \frac{r_1}{r_2} \right)^2 |R^*(\omega)|^2 e^{2j\omega \tau} + \cdots \]

In the time domain, equation (6) is:

\[ p'(t) = p_1'(t) + \frac{r_1}{r_2} h(t - \tau) + \frac{r_1}{r_2} h(-t + \tau) - \frac{1}{2} \left( \frac{r_1}{r_2} \right)^2 h(t - \tau) \ast h(t - \tau) - \frac{1}{2} \left( \frac{r_1}{r_2} \right)^2 h(-t + \tau) \ast h(-t + \tau) + \cdots \]

where \( p'(t) \) is the power cepstrum of the total pressure signal, and \( p_1'(t) \) is the power cepstrum
of the direct pressure signal. Retracing the steps above, the power cepstrum is defined as the
inverse fourier transform of the natural logarithm of the square of the pressure spectrum
amplitude.

Comparing equations (1) and (7) shows that both equations contain the desired quantity, \( h(t) \); however, to extract \( h(t) \) using equation (1) it is necessary for the direct signal and the
reflected signal do not overlap. For most practical measurement arrangements, the direct
signal could only be a few milliseconds long. With this short signal, it would be difficult to
obtain a good signal-to-noise ratio, especially at low frequency. To extract \( h(t) \) from equation
(7), it is only necessary that the power cepstrum of the direct pressure signal does not overlap
the power cepstrum of the reflected pressure signal. This condition can easily be met using
the procedures described later in this paper for minimizing the cepstrum length.

In Bolton’s original work [1], the direct signals were 10ms long because his analog-to-digital
converter could only store 4096 points. The system described here typically uses signals
several seconds (131,072 points) long, so the signal-to-noise ratio is improved especially at
low frequency.

3. Method

The method described here is a combination of techniques originally applied by Bolton [1]
and new techniques (low frequency data synthesis and high frequency matching). The
method is described below.

3.1 Signal Generation

The signal to be played by the loudspeaker is produced by modifying a swept sine signal to
have minimum phase (and thus minimum length impulse response).

The swept sine signal is generated using the following equation:

\[
s(t) = \sin(at^2 + bt),
\]

where \( a = 2\pi(f_2 - f_1)/2t_s \), \( b = 2\pi f_1 \), \( f_1 \) and \( f_2 \) are the starting and ending frequency in Hz,
and \( t_s \) is the signal duration in seconds.

The swept sine signal is not optimized for the cepstrum technique because its spectrum has
ripples which require more terms (length) of the direct arrival cepstrum, \( p_1(t) \). The
minimum-phase-signal algorithm [2] is used to optimize signal. The minimum phase signal
has uniform spectral amplitude and optimized phase to produce the shortest possible
cepstrum signal.

3.2 Processing the Acquired Signal

3.2.1 Time Domain Averaging

During data acquisition, the signal is time domain averaged to reduce background noise not
correlated to the test signal. Stationary noise (e.g. air conditioning system noise) is reduced
by 10 dB each time the number of samples is increased by ten times. Non-stationary noise
(e.g. a door closing) is reduced by 20 dB for increasing the sampling by ten times. The time domain averaging requires that the data acquisition be precisely triggered.

3.2.2 Late Echo Removal
Testing in a non-anechoic environment introduces undesirable echoes from the walls, etc. These are removed by: (1) taking the DFT of the signal and setting the phase equal to zero; (2) taking the IDFT (inverse discrete Fourier transform) to obtain the impulse response of the entire measurement system; (3) windowing the impulse response to remove echoes occurring after the reflection from the sample; and (4) taking the DFT to obtain the spectrum with the late echoes removed. It is important to use a window function with a smooth transition.

3.2.3 High Frequency Matching
The transition at the Nyquist frequency of the frequency spectrum must be smooth to minimize cepstrum noise. This transition is made smooth by removing equal numbers of data points on each side of the Nyquist frequency until the transition is smooth. Note that after this operation the number of points in the spectrum is not a radix-2 number, so the more general DFT must be used for processing rather than the more common FFT (fast Fourier transform).

3.2.4 Low Frequency Synthesis
The transition between sample frequency and zero frequency of the spectrum must be smooth to minimize cepstrum noise. No measurement system can have perfect frequency response to zero frequency, so the inevitable roll off in system response at low frequency is undesirable. The low frequency spectrum is synthesized by assuming that at low frequency the sample has no absorption (a good assumption for the measurement system here where the data is synthesized below 20 Hz). The signal spectrum calculated assuming perfect reflection from the sample is substituted for the measured signal spectrum. It is important to have a smooth transition between the synthesized data and the measured data to minimize cepstrum noise.

3.2.5 Power Cepstrum Windowing
After the measured spectrum has been improved as described above, the power cepstrum is calculated by taking the IDFT of the natural logarithm of the square of the pressure spectrum amplitude. The cepstrum is windowed to remove all the terms not associated with the reflection from the sample, \((r_1/r_2)h(t-\tau)\). The transitions of the windowing function must be smooth to minimize corruption of the reflection coefficient. The DFT is performed on the reflection term, and the result is scaled by \((r_2/r_1)\) to obtain the reflection coefficient, \(R(\omega)\).

4. Hardware Implementation
A diagram of the hardware setup is shown in Figure 1.

The following hardware issues are of note:

(1) A high quality soundcard should be used.
(2) A monitor quality coaxial loudspeaker is used because the coaxial design is consistent with the theoretical description \(\phi/\tau\), loudspeaker, microphone, and material in line). Possible errors associated with using a more conventional two-way loudspeaker were not
investigated. The loudspeaker must have a broad frequency spectrum with very little ripple.

(3) A type 1 quality microphone and amplifier was used. Early attempts to use a lower quality microphone (piezoelectric) produced degraded results. The microphones and associated amplifiers were the most expensive hardware items.

The overall goal of the hardware implementation was to minimize noise and to maximize the smoothness of the equipment frequency response. Minimum noise is important, because the power cepstrum operates on the logarithm of the measured spectrum which makes it very sensitive to noise. Maximizing the smoothness of the equipment frequency response reduces the length of the direct-arrival power cepstrum. Direct-arrival-power-cepstrum information that overlaps the reflection information has the same effect as noise.

5. Comparison with Impedance Tube Results

Figure 2 provides a comparison of cepstrum-technique and impedance-tube results for several typical automotive-carpet-backing materials. The system described here has been implemented as a tool for comparing the relative absorption of samples, and the agreement is adequate for that purpose. However, more work is required for the system to provide precise sound absorption results.

The sources of error are:

(1) Due to the compact nature of the portable system, there is some overlap of the cepstrum of the direct signal and the desired reflection cepstrum term (i.e. cepstral noise).
(2) Even with sample averaging and in a relatively quiet laboratory environment, noise is significant in the measurement.
(3) The sample size (nominally 1m by 1m) is relatively small which causes the desired reflection cepstrum term to sometimes overlap the edge refraction.

The system is optimized for its current duties for quick non-destructive comparisons of samples or components in a lab or office environment. The measurement would be improved
by increasing the scale of the setup (loudspeaker, microphone, sample spacing) and making
the measurement in a very quiet anechoic chamber.

Figure 2: Comparison of Cepstrum and Impedance Tube Measurement Results.

6. Conclusion

A methodology and measuring system for measuring the sound absorption of automotive
carpet samples in an office environment is described. The results show reasonable agreement
with impedance tube measurements, and the results are adequate for comparisons between
similar samples.

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